Optimal transport and mathematical finance: the geometry of model-uncertainty

Abstract

Mathematical models of financial markets are ubiquitously used in the banking industry and form the basis for accounting, regulation, and supervision. Whenever one produces the annual balance sheet of a financial institution, mathematical models are used to assign a cash value to the various assets and liabilities, to determine the capital requirement, etc. A fundamental problem is model-uncertainty: there is no such thing as a single correct model. Which type of model properly describes the market's behaviour depends on the purpose and varies over time. Only in hindsight one can evaluate whether a particular model was appropriate. It is a challenging problem to quantify the effects of model-uncertainty which is notoriously neglected in practice: Starting with the "Black Monday" 1987 and down to the present day, the over-confidence in mathematical models and the failure to account for model-uncertainty have their share in the respective financial crises. The remedy we envision is to provide a systematic method to estimate and quantify the consequences of model-uncertainty. Given a concrete financial derivative and a specific financial market we need to under- stand how the "predicted" value of the derivative varies as different possible models are considered. Specifically, we need to address the following fundamental challenges: Goal 1 - absolute model-uncertainty: characterize the worst case models in a specific setup, calculate the range of corresponding values for a given derivative. Goal 2 - model-uncertainty with a prior: Given a plausible reference model, how sensitive is the corresponding value of a specific derivative with respect to changes of the reference model? The main goal of the proposed project is to provide satisfactory answers to these questions inspired by the classical theory of Monge-Kantorovich optimal mass transport. A fundamental recent insight in the finance literature (see [5, 12, 11, 8] among many others) is that a probabilistic analogue of Kantorovich's duality theory can be established for the problem of model-uncertainty. As a consequence, the economic problem can be rigorously linked to a probabilistic version of the optimal transport problem. To resolve the problem of model-uncertainty it is necessary to build a systematic theory for this probabilistic transport problem. We shall aim for the pace of the spectacular mathematical developments which classical optimal transport has seen in the recent decades. These developments started in the 1990s with Gangbo-McCann's [13] insight that optimal transport plans can be characterized in geometric terms and Benamou-Brenier's [7] continuous time formulation of the transport problem, leading to Otto's [20] transport based notion of a differentiable calculus. Today the field is famous for its striking applications in areas ranging from mathematical physics and PDE-theory to geometric and functional inequalities. While the probabilistic setup required in the financial context seems a priori rather different from the classical transport problem, recent contributions [4, 6, 3, 18] of the PI and his collaborators establish probabilistic counterparts of the foundational results [13, 7]. The results obtained in [4, 6] demonstrate that the geometrical approach of Gangbo-McCann can be developed into an analogous apparatus in the probabilistic context with an immediate impact on some classical problems of stochastic analysis. In particular it will enable us to provide concise geometric descriptions of worst case models, allowing to systematically address Goal 1. In [3, 18] we develop a stochastic extension of the Benamou-Brenier transport formulation, identifying a natural geometry on the space of models that will allow us to develop the differential calculus required to tackle Goal 2. Together, these results will form the main pillars of a probabilistic counterpart to the achievements in classical transport, leading to a systematic theory of model-uncertainty. While model-uncertainty is the main focus of the proposed project, the envisioned theory of probabilistic optimal transport has a much wider potential, which will be exploited in a number of further naturally related topics: the theory of insider trading (cf. [1]), enlargement of filtrations, the Schroedinger problem, functional inequalities, and potential games to mention a few. Our hope is that the proposed project can play an important role in initiating these anticipated developments.

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101007 - Mathematical Finance (40%) | 101019 - Stochastics (30%) | 101016 - Optimisation (30%)

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Further links about the involved persons and regarding the project you can find at
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